CHAPTER 4

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Use matrices and determinants in real-life situations. **(4.1–4.5)**	Organize data, such as the number and dollar value of Hispanic music products shipped. **(p. 204)**
Perform matrix operations.	
• add and subtract matrices **(4.1)**	Find the total cost of college tuition plus room and board. **(p. 205)**
• multiply a matrix by a scalar **(4.1)**	Write U.S. population data as percents of total population. **(p. 205)**
• multiply two matrices **(4.2)**	Calculate the cost of softball equipment. **(p. 210)**
Evaluate the determinant of a matrix. **(4.3)**	Find the area of the Golden Triangle. **(p. 220)**
Find the inverse of a matrix. **(4.4)**	Encode or decode a cryptogram. **(pp. 225, 226)**
Solve matrix equations. **(4.1, 4.4, 4.5)**	Extend the process of equation solving to equations whose solutions are matrices. **(pp. 201, 224)**
Solve systems of linear equations.	
• using Cramer's rule **(4.3)**	Find the cost of gasoline. **(p. 220)**
• using inverse matrices **(4.5)**	Calculate a budget. **(p. 235)**
Use systems of linear equations to solve real-life problems. **(4.3, 4.5)**	Decide how much money to invest in each of three types of mutual funds. **(p. 232)**

How does Chapter 4 fit into the BIGGER PICTURE of algebra?

Your study of linear algebra has continued with Chapter 4. Matrices are used throughout linear algebra, especially for solving linear systems. This introduction to matrices, their uses, and properties of matrix operations also connects to your past. For example, instead of multiplying real numbers, or variables that represent real numbers, you multiplied matrices. You also saw properties, such as the commutative property of multiplication, that apply to real numbers but not to matrices.

STUDY STRATEGY

Did you write out the steps?

Here is an example of several steps you can write when multiplying two matrices, following the **Study Strategy** on page 198.

Writing Out the Steps

$$\begin{bmatrix} 1 & -6 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -9 & 0 \\ -2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-9) + (-6)(-2) & 1(0) + (-6)7 \\ -3(-9) + 2(-2) & -3(0) + 2(7) \end{bmatrix}$$

$$= \begin{bmatrix} -9 + 12 & 0 - 42 \\ 27 - 4 & 0 + 14 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -42 \\ 23 & 14 \end{bmatrix}$$

CHAPTER 4 Chapter Review

VOCABULARY

- matrix, p. 199
- dimensions of a matrix, p. 199
- entries of a matrix, p. 199
- row matrix, p. 199
- column matrix, p. 199
- square matrix, p. 199
- zero matrix, p. 199
- equal matrices, p. 199
- scalar, p. 200
- determinant, p. 214
- Cramer's rule, p. 216
- coefficient matrix, p. 216
- identity matrix, p. 223
- inverse matrix, p. 223
- matrix of variables, p. 230
- matrix of constants, p. 230

4.1 MATRIX OPERATIONS

Examples on pp. 199–202

EXAMPLES You can add or subtract matrices that have the same dimensions by adding or subtracting corresponding entries.

$$\begin{bmatrix} 5 & -2 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 1 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 5+9 & -2+1 \\ 0+(-4) & 6+4 \end{bmatrix} = \begin{bmatrix} 14 & -1 \\ -4 & 10 \end{bmatrix}$$

You cannot subtract these matrices because they have different dimensions.

$$\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 1 & -5 & -4 \\ 2 & 7 & 1 \end{bmatrix}$$

To do scalar multiplication, multiply each entry in the matrix by the scalar.

$$-3\begin{bmatrix} -12 & -6 \\ 3 & 1 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} (-3)(-12) & (-3)(-6) \\ (-3)(3) & (-3)(1) \\ (-3)(2) & (-3)(8) \end{bmatrix} = \begin{bmatrix} 36 & 18 \\ -9 & -3 \\ -6 & -24 \end{bmatrix}$$

To solve this matrix equation, equate corresponding entries and solve for x and y.

$$\begin{bmatrix} x+2 & 2 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ -1 & 3y \end{bmatrix} \qquad \begin{aligned} x+2 &= -6 \\ x &= -8 \end{aligned} \qquad \begin{aligned} 3y &= 9 \\ y &= 3 \end{aligned}$$

Perform the indicated operation if possible. If not possible, state the reason.

1. $\begin{bmatrix} 15 & 4 \\ 3 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 2 & 7 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

3. $\begin{bmatrix} 6 & 10 \\ 9 & 6 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 7 \\ 4 & 7 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 1 & 5 \\ -2 & 3 & 1 \\ 1 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 4 & 1 \\ 2 & -3 \end{bmatrix}$

5. $2\begin{bmatrix} 4 & 6 & -1 \\ 10 & -5 & 2 \\ 0 & 11 & 1 \end{bmatrix}$

6. $\frac{1}{2}\begin{bmatrix} -2 & 0 \\ 4 & 8 \\ -6 & -2 \end{bmatrix}$

Solve the matrix equation for *x* and *y*.

7. $\begin{bmatrix} 1 & 14 \\ -5x & 10 \end{bmatrix} = \begin{bmatrix} y-9 & 14 \\ 5 & 10 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 4y \\ -1 & 13 \end{bmatrix} + \begin{bmatrix} -6 & 5 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -7 \\ x & 13 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 3y \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -4 \\ x & -2 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 3 & -3 \end{bmatrix}$

10. $\begin{bmatrix} 7y & -2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ x & -3 \end{bmatrix} = \begin{bmatrix} 6 & -7 \\ -2 & 8 \end{bmatrix}$

4.2 MULTIPLYING MATRICES

Examples on pp. 208–210

EXAMPLE You can multiply a matrix with n columns by a matrix with n rows.

$$\begin{bmatrix} -6 & 1 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (-6)(6) + (1)(0) & (-6)(3) + (1)(1) \\ (5)(6) + (-2)(0) & (5)(3) + (-2)(1) \end{bmatrix} = \begin{bmatrix} -36 & -17 \\ 30 & 13 \end{bmatrix}$$

Write the product. If it is not defined, state the reason.

11. $\begin{bmatrix} 12 \\ -4 \end{bmatrix}\begin{bmatrix} -10 & -7 \end{bmatrix}$

12. $\begin{bmatrix} 2 & 15 \\ -3 & 10 \end{bmatrix}\begin{bmatrix} -5 & 12 \\ 1 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 7 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 3 & -1 & 8 \\ 2 & -4 & 8 \end{bmatrix}$

4.3 DETERMINANTS AND CRAMER'S RULE

Examples on pp. 214–217

EXAMPLES You can evaluate the determinant of a 2×2 or a 3×3 matrix. Find products of the entries on the diagonals and subtract.

$$\det \begin{bmatrix} -2 & -6 \\ 1 & 4 \end{bmatrix} = \begin{vmatrix} -2 & -6 \\ 1 & 4 \end{vmatrix} = -2(4) - 1(-6) = -8 + 6 = -2$$

$$\det \begin{bmatrix} 2 & 1 & 5 \\ -1 & 6 & 3 \\ 2 & -4 & 2 \end{bmatrix} = \begin{vmatrix} 2 & 1 & 5 \\ -1 & 6 & 3 \\ 2 & -4 & 2 \end{vmatrix} \begin{matrix} 2 & 1 \\ -1 & 6 \\ 2 & -4 \end{matrix} = (24 + 6 + 20) - [60 + (-24) + (-2)] = 16$$

You can find the area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) using

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where $\pm$ indicates you should choose the sign that yields a positive value.

You can use Cramer's rule to solve a system of linear equations. First find the determinant of the coefficient matrix and then use Cramer's rule to solve for x and y.

$3x - 4y = 12$
$x + 2y = 14$

$$\det \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3(2) - 1(-4) = 6 + 4 = 10$$

$$x = \frac{\begin{vmatrix} 12 & -4 \\ 14 & 2 \end{vmatrix}}{10} = \frac{12(2) - 14(-4)}{10} = \frac{80}{10} = 8 \qquad y = \frac{\begin{vmatrix} 3 & 12 \\ 1 & 14 \end{vmatrix}}{10} = \frac{3(14) - 1(12)}{10} = \frac{30}{10} = 3$$

Evaluate the determinant of the matrix.

14. $\begin{bmatrix} -9 & 1 \\ 3 & 2 \end{bmatrix}$

15. $\begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$

16. $\begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix}$

17. $\begin{bmatrix} 2 & -3 & 4 \\ 0 & 1 & -2 \\ 1 & 2 & -3 \end{bmatrix}$

18. Find the area of a triangle with vertices $A(0, 1)$, $B(2, 4)$, and $C(1, 8)$.

Use Cramer's rule to solve the linear system.

19. $7x - 4y = -3$
$2x + 5y = -7$

20. $2x + y = -2$
$x - 2y = 19$

21. $5x - 4y + 4z = 18$
$-x + 3y - 2z = 0$
$4x - 2y + 7z = 3$

4.4 Identity and Inverse Matrices

Examples on pp. 222–226

EXAMPLES You can find the inverse of an $n \times n$ matrix provided its determinant does not equal zero.

The inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

If $A = \begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$, then $A^{-1} = \frac{1}{7(2) - 5(3)}\begin{bmatrix} 2 & -3 \\ -5 & 7 \end{bmatrix} = -1\begin{bmatrix} 2 & -3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix}$.

You can use the inverse of a matrix A to solve a matrix equation $AX = B$: $X = A^{-1}B$.

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}X = \begin{bmatrix} 3 & 0 \\ 5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7 - 6}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -1 & 2 \end{bmatrix}$$

Find the inverse of the matrix.

22. $\begin{bmatrix} 2 & 3 \\ 7 & 11 \end{bmatrix}$ **23.** $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ **24.** $\begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$ **25.** $\begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$

Solve the matrix equation.

26. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}X = \begin{bmatrix} 0 & 9 \\ -1 & 4 \end{bmatrix}$ **27.** $\begin{bmatrix} -7 & -5 \\ 4 & 3 \end{bmatrix}X + \begin{bmatrix} 8 & -2 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 6 & 2 \end{bmatrix}$

4.5 Solving Systems Using Inverse Matrices

Examples on pp. 230–232

EXAMPLE You can use inverse matrices to solve a system of linear equations.

$x + 3y = 10$
$2x + 5y = -2$

Write in matrix form. $\underset{A}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 10 \\ -2 \end{bmatrix}}$

Then $X = A^{-1}B = \frac{1}{1(5) - 2(3)}\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \end{bmatrix} = -1\begin{bmatrix} 56 \\ -22 \end{bmatrix} = \begin{bmatrix} -56 \\ 22 \end{bmatrix}$.

The solution is $(-56, 22)$.

Use an inverse matrix to solve the linear system.

28. $9x + 8y = -6$
$-x - y = 1$

29. $x - 3y = -2$
$5x + 3y = 17$

30. $4x - 14y = -15$
$18x - 12y = 9$

Use an inverse matrix and a graphing calculator to solve the linear system.

31. $x - y - 4z = 3$
$-x + 3y - z = -1$
$x - y + 5z = 3$

32. $4x + 10y - z = -3$
$11x + 28y - 4z = 1$
$-6x - 15y + 2z = -1$

33. $5x - 3y + 5z = -1$
$3x + 2y + 4z = 11$
$2x - y + 3z = 4$

CHAPTER 4

Chapter Test

Perform the indicated operation(s).

1. $\begin{bmatrix} 2 & 5 & -4 \\ 3 & 0 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 7 \\ -2 & -5 & 7 \end{bmatrix}$

2. $0.25\begin{bmatrix} 8 & 20 & -12 \\ -8 & -4 & 36 \end{bmatrix}$

3. $-4\left(\begin{bmatrix} 1 & 10 \\ -4 & -6 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -3 & -8 \end{bmatrix}\right)$

4. $\begin{bmatrix} 4 & 1 & 4 \\ -1 & 8 & -3 \\ 4 & 3 & 0 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix}$

5. $\begin{bmatrix} -6 & 1 \\ 9 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -5 & 4 \end{bmatrix}$

6. $\begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ 4 & 6 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Solve the matrix equation for *x* and *y*.

7. $\begin{bmatrix} -1 & y+6 \\ x-4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ -9 & 3 \end{bmatrix}$

8. $\begin{bmatrix} -22 & 9 \\ 1 & -y \end{bmatrix} = \begin{bmatrix} 2x & 9 \\ 1 & 4 \end{bmatrix}$

9. $3\begin{bmatrix} x & 1 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} -15 & 3 \\ y & -12 \end{bmatrix}$

Evaluate the determinant of the matrix.

10. $\begin{bmatrix} 7 & -9 \\ -3 & 4 \end{bmatrix}$

11. $\begin{bmatrix} -2 & -1 \\ 1 & -1 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 0 & 1 \\ 1 & 5 & 3 \\ 2 & 2 & 0 \end{bmatrix}$

13. $\begin{bmatrix} -1 & 3 & 4 \\ 6 & 0 & -2 \\ 0 & -5 & 1 \end{bmatrix}$

Find the area of the triangle with the given vertices.

14. $A(2, 1), B(5, 3), C(7, 1)$

15. $A(-1, 0), B(-3, 3), C(0, 4)$

16. $A(-3, 2), B(-1, 4), C(-4, 3)$

Use Cramer's rule to solve the linear system.

17. $2x + y = 12$
$5x + 3y = 27$

18. $-4x + 5y = -10$
$5x - 6y = 13$

19. $x + y = 2$
$2y - z = 0$
$-x - y + z = -1$

20. $5x - 2y + 7z = 12$
$2x + 5y + 3z = 10$
$3x - y + 4z = 8$

Find the inverse of the matrix.

21. $\begin{bmatrix} 4 & 5 \\ 3 & 9 \end{bmatrix}$

22. $\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

23. $\begin{bmatrix} -6 & 4 \\ 6 & -5 \end{bmatrix}$

24. $\begin{bmatrix} 1 & 0 \\ 0 & -5 \end{bmatrix}$

Solve the matrix equation.

25. $\begin{bmatrix} 8 & 7 \\ 1 & 1 \end{bmatrix}X = \begin{bmatrix} 3 & -6 \\ -2 & 9 \end{bmatrix}$

26. $\begin{bmatrix} 2 & 5 \\ 2 & 6 \end{bmatrix}X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

27. $\begin{bmatrix} 1 & 0 \\ -6 & 2 \end{bmatrix}X = \begin{bmatrix} 10 & 6 & 8 \\ 4 & 12 & 2 \end{bmatrix}$

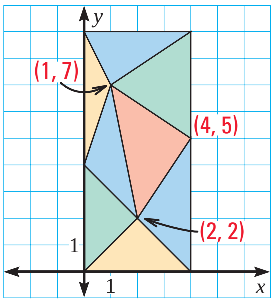
Use an inverse matrix to solve the linear system.

28. $x - y = 5$
$-2x + 3y = -9$

29. $3x + 2y = -8$
$-2x + 5y = 18$

30. $2x - 7y = 6$
$-3x + 11y = -10$

31. **STAINED GLASS** You are making a stained glass panel using different colors as shown. The coordinates given are measured in inches. Find the area of the red triangle.



32. **DECODING** Use the inverse of $A = \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix}$ and the coding information on pages 225 and 226 to decode the message below.

44, –15, 3, –1, 80, –32, 39, –17, 3, –1, 12, –4, 77, –26

33. **BUDGETING MEALS** You have \$18 to spend for lunch during a 5 day work week. It costs you about \$1.50 to make a lunch at home and about \$5 to buy a lunch. How many times each work week should you make a lunch at home?